AP Calc AB Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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 WS Assessment

 Target 4:

Infinite limits and

Intermediate value theorem (iVT)

**I can:**

* Interpret the behavior of functions using limits involving infinity
* Explain the behavior of a function on an interval using the Intermediate Value Theorem

Unit 1: Limits and Continuity

HW Target 4

Unit 1 Progress Check FRQ Part A and Part B

The symbol for infinity (∞) does not represent any real number. When we say “the limit of f as x approaches infinity” we mean the limit of function f as x moves increasingly far to the right on the number line.

 Facts: When the *denominator* approaches zero then the *fraction* approaches infinity

 When the *denominator* approaches infinity then the *fraction* approaches zero

$\lim\_{x\to +\infty }\frac{1}{x}=?$ $\lim\_{x\to -\infty }\frac{1}{x}=?$ We say $\lim\_{x\to \infty }\frac{1}{x}=?$

We say the line y = 0 is a **horizontal asymptote** of the graph f(x) = 1/x. Check with calculator.

Prove the horizontal asymptote of $f(x)=1+\frac{1}{x}$ is the line y = 1

(Both algebraic and desmos, stamp)

Prove the vertical asymptote of $f(x)=x+\frac{1}{x}$ is the line x = 0. Hint: both sides

(Both algebraic and desmos, stamp)

We say the line x = a is a **vertical asymptote** of the graph f(x) if either

 $\lim\_{x\to a^{+.}}f\left(x\right)=\pm \infty $or $\lim\_{x\to a^{-.}}f(x)=\pm \infty $

Identify all asymptotes of $f\left(x\right)=\frac{x}{\sqrt{x^{2}-1}} $ i.e finding limit $\lim\_{x\to \pm \infty }f(x)$,$\lim\_{ x\to a^{\pm }}f\left(x\right)=\pm \infty $

Identify all asymptotes of $f\left(x\right)=\frac{1+e^{x}sin⁡(x)}{e^{x-1}-1} $ . Sketch graphs

Find the following limit and state its corresponding asymptotes. You may sketch its graph to support your answer. Hint: both sides

$\lim\_{x\to 0}\frac{1}{x}$ $\lim\_{x\to \infty }\frac{1}{x} $ $\lim\_{x\to \infty }(1+\frac{1}{x})$ $\lim\_{x\to 0}(1+\frac{1}{x})$

Identify all asymptotes of the following functions

a. f(x) = $\frac{-15x}{7x+4}$ b. $f\left(x\right)=\frac{5x^{2}+8x-3}{3x^{2}+2}$

c. f(x) =$\frac{11x+2}{2x^{3}-1}$ d. $ f\left(x\right)=\frac{2x^{2}-3}{7x+4}$

e. f(x) = $\frac{-4x^{3}+7x}{2x^{2}-3x-10}$ f. f(x) =$\frac{-4x^{3}+7x}{2x^{2}-3x-10}$

The Intermediate Value Theorem for Continuous Functions:

*If f(x ) is continuous on the interval [a,b] with f(a)* $\ne $ *f(b), then if d is any number between f( a) and f( b), there is at least one c between a and b such that f(c)= d .*

More simply put, provided f is continuous on [a,b] and f(a) ≠ f(b), then every number between the output at a and the output at b is also an output for f.

In other words, if yo is between f(a) and f(b) then yo = f(c) for some c in [a, b]

 (When the function is continuous, it has no gap)

In each of the following, decide whether the Intermediate Value Theorem (IVT) can or cannot be used to justify the stated conclusion. Then explain why or why not.





Is any real number exactly 1 less than its cube? i.e. Prove that x = x3 – 1 has solution

We answer this by applying the Intermediate Value Theorem (IVT) to check whether

* the function f(x) = x3 – x – 1 is continuous
* when the function has its zero value.

 Here is the graph of the function = Polynomial

 We see the function has its zero between x = 1 and x = 2

 We say: *A function y = f(x) that is continuous on a a closed interval*

 *[\_\_, \_\_] takes on every value between f(\_\_ ) and f(\_\_ ).*

 In other words, **if** zero (0) is between f(\_\_) and f(\_\_\_) **then**

0 = f(c) for some c in [\_\_\_, \_\_\_\_\_] Find real value of c = ? \_\_\_\_ (3 decimal)

Is any real number exactly 1 less than its fourth power? Your writing should include the IVT’s wording.



Assessment

Find the following limits

 $\lim\_{x\to 0}\frac{sin(3x)}{sin(4x)}$ $\lim\_{x\to 3^{+.}}\frac{\sqrt{t^{2}-9}}{t-3}$ $\lim\_{x\to 0}\frac{1-cosx}{sin^{2}x}$

Identify all asymptotes of the following functions

 f(x) =$\frac{x^{2}-x}{x^{2}-6x+5}$ f(x) =$\frac{e^{-x}}{1-e^{-x}}$

Decide whether the Intermediate Value Theorem (IVT) can or cannot be used to justify the stated conclusion. Then explain why or why not.



Is any real number exactly 2 more than its cube? Give any such values accurate to 3 decimal places.